Art and science of mathematical medals

João Bernardo dos Santos

Fig 1. Illustration from De Humani Corporis Fabrica Libri Septem by Andreas Vesalius Photo: Public Domain

With the XXXVII FIDEM Congress being held in Florence, birthplace of European Renaissance, the marriage of Art and Science felt a most appropriate topic. Thusly, evoking the Renaissance Spirit, and drawing on the personal experience of studying both medal-making and mathematics, this lecture intends to bring both fields together. We will identify similarities between the artist and the mathematician, and argue in favour of mathematics having inherent aesthetic properties that can be expressed through the art medal paradigm, with examples of contemporary Portuguese medallic artwork.

Leonardo da Vinci is a prime example of the Renaissance polymath – and Florentine in origin, nonetheless – but as further immersion in this context, I recall the legacy of Andreas Vesalius (1514–1564). The renowned physician – a revolutionary in that field – authored *De Humani Corporis Fabrica Libri Septem* (1543), which is still a remarkable work, of both scientific and artistic merit. The anatomical drawings[1] show the bodies in action, the skeleton and muscle groups in natural or allegorical poses, and not without symbolic allusions (fig. 1). Their study informs both scientific and artistic values.

Is such a marriage possible in contemporary art medals? Could the most exact of the sciences have some common ground with the arts, the most subjective of fields?

We shall direct our attention firstly to the practitioners of these fields. The mathematician and the artist have more in common than quick judgement might suggest. And these similarities are of disposition, purpose, drive, and values.

The mathematician's palette is covered by an array of axioms and theorems, instead of paints. But having mastered a number of techniques, their most important tool is the creativity with which they mix and match what is known, to come up with new ideas, approaches, and patterns. The pure mathematician is not driven by utility and practical applications, as mathematician David Hilbert proclaimed in his retirement lecture in 1930 (Vinnikov, 1999). Such things as the building of better bridges or the orbits of GPS satellites do not motivate them and are mere consequence of mathematical research. What moves the mathematician is a pulsating curiosity, a need to know. As Hilbert's epitaph summarizes, *Wir müssen wissen, Wir werden wissen*[2]. If a mathematical statement is proved, it is so for all time. Hence, mathematics is a pursuit of universality, truth, and – perhaps surprisingly – beauty. Is there really beauty in such an abstract and mental subject? As we will see in a moment, mathematicians will testify accordingly. Elegance, for instance, is a frequent concept referred by these practitioners. And in the words of Leonardo (Da Vinci, 1817, p. 29), Painting also has a mental and – in the sense of his time – scientific nature.

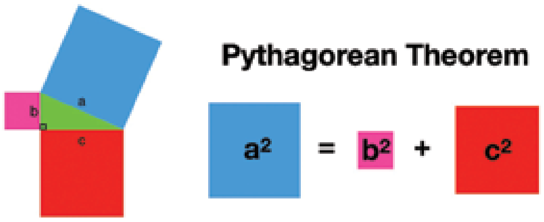

Fig 2. Diagram of Pythagorean Theorem
Photo: João Bernardo dos Santos

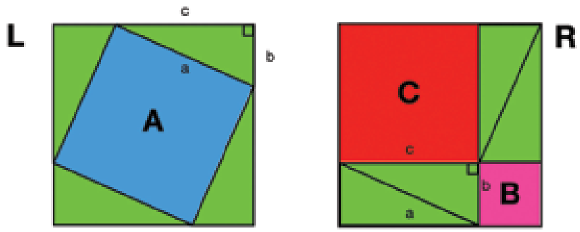

Fig 3. Construction for the demonstration of the Pythagorean Theorem
Photo: João Bernardo dos Santos

Fig 4. Português sem Mestre, 2022
José João Brito
Oxydized brass, fabricated, 105 x 100 x 50 mm
Photo: José João Brito

In his 1940 essay *A Mathematician's Apology*, G. H. Hardy wrote of the utmost importance of curiosity in the mathematical work, as he experienced the waning of his own. And admitting difficulty in defining mathematical beauty – much like any beauty is hard to define – he affirms it is essential. 'No permanent place in the world for ugly mathematics' (Hardy, [1940] 2005, p. 14).

Bertrand Russell describes mathematics as not only possessing truth, but 'supreme beauty – a beauty cold and austere, like that of sculpture' (Russell, [1907] 1919, p. 60). He denotes experiences of exaltation and of the sublime, and the 'sense of being more than man' in mathematics such as in poetry, referring the importance of individual practice in drawing by hand as a mean for true comprehension (Russell, [1907] 1919, pp. 60-62).

The mathematician, in parallel with the artist, faces his blank sheet of paper, moved by the impulse to create and discover – and, as Hardy ([1940] 2005, p. 11) unapologetically notes, the desire for personal success and recognition – valuing clever use of the means available, a degree of unexpectedness, mastery of technique, elegance, structure, connection, and meaningfulness.

The common mathematics education promotes blind acceptance of seemingly arbitrary and incomprehensible rules. Bertrand Russell's description of this phenomenon in his 1907 essay *The Study of Mathematics* (Russell [1907] 1919, pp. 62-64) still rings true today. Omitted from many classrooms, thusly preventing the students from encountering mathematical beauty and true understanding, the mathematical proof is a great source of intellectual pleasure and aesthetic experience. As we turn – in the arts – to Ancient Greece for examples of classical beauty, so shall we for mathematical beauty, by revisiting an old acquaintance.

The Pythagorean Theorem (fig. 2) has been around for millennia, and has been proven in hundreds of different ways, over the years. We will see a rather visual demonstration of why it is true, before turning our attention to the medal world.

In mathematics, to prove a statement to be true, no finite quantity of true examples is enough. A big number of right triangles verifying the theorem can make us suspect it to be true, but should never satisfy us. To prove this for every single right triangle, we will take one with the sides a, b, and c, which stand for any value for the sides. If proven for this general triangle, it will be proven for all examples.

As shown in figure 3, we will start by making copies of our right triangle and arrange them so they make the square L, of side $b + c$. The area of L – which is equal to $(b + c) \times (b + c)$ – is composed by the areas of the four triangles and the remaining space, also a square, which we will name A.

Now, we make a copy of square L, but we rearrange the triangles inside. We will call this new square R.

The area of square R is the same as L – still $(b + c) \times (b + c)$ – but the space not occupied by the four triangles is now divided in two smaller squares. Let them be B, and C.

As the area of squares L and R is the same, and so are the areas of the four triangles, the area of the remaining space must also be the same. That is to say, $area_A = area_B + area_C$.

Now, A is a square of side a, so $area_A = a \times a = a^2$. Following the same logic, $area_B = b \times b = b^2$, and $area_C = c \times c = c^2$.

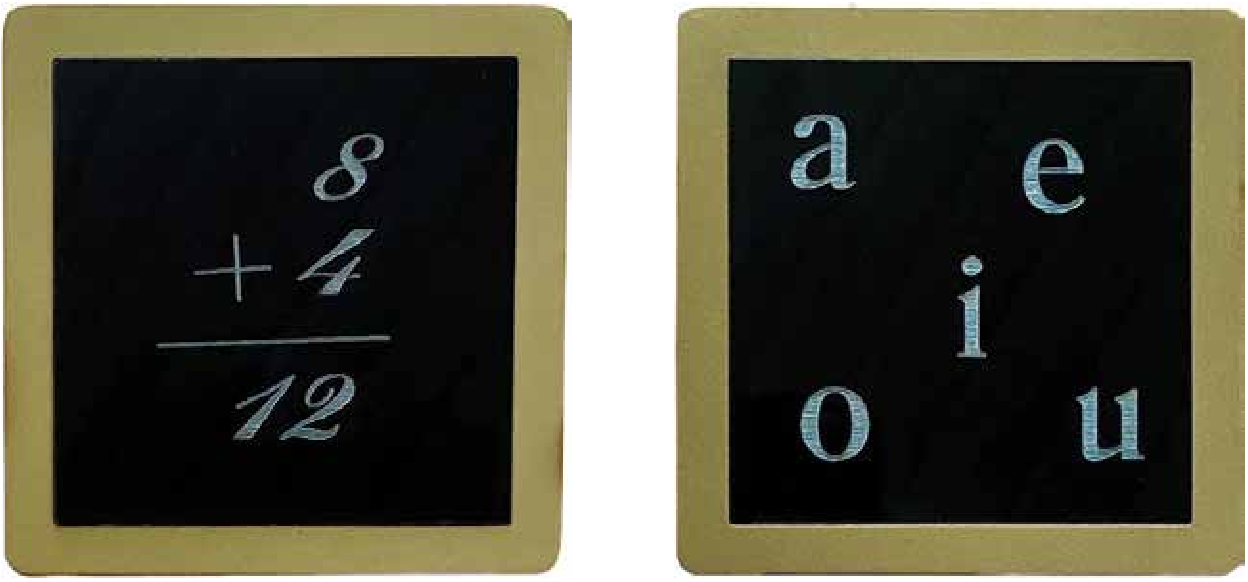

Fig 5. Primeiras letras, [date unlnown]
Hélder Batista
Brass and acrylic glass, fabricated, 70 x 70 x 3 mm
Collection: GRAVARTE
Photo: João Bernardo dos Santos

Fig 6. 125.º Aniversário do Banco de Portugal, 1971
António Duarte
Bronze, struck, 80 mm
Photo: João Bernardo dos Santos

Therefore, since $area_A = area_B + area_C$, and $area_A = a^2$, $area_B = b^2$, and $area_C = c^2$, we finally prove $a^2 = b^2 + c^2$.

If there was success in presenting this demonstration in a clear way, at least some of the readers will have experienced the sense of satisfaction of a mathematical aesthetic experience, as it has been described so far.

Medals can and have brought the abstract beauty of maths to the material world (figs. 4-6). From a simple summation algorithm that brings back times gone by, to platonic references of profound significance and symbolism, there is a vast field of possible interactions between pure mathematics and art medals.

The Golden Ratio, or divine proportion, is a popular subject in both mathematics and the arts. But its prevalence in both the natural world and the arts has been greatly overestimated (Freitas, 2020). It is the irrational number

$$\frac{1+\sqrt{5}}{2} \approx 1.618 \ldots,$$

commonly attributed the Greek letter φ (phi), as an homage to the ancient Greek sculptor Phidias, frequently believed to have used the golden ratio is his work, which is not the case. It is neither present in the Parthenon, nor in the Vitruvian Man, the Eiffel Tower, or even the shell of the Nautilus, one of the most cited occurrences of the golden spiral (Freitas, 2020). But it is present in the seeds of a sunflower, architectural work of Le Corbusier and artwork of Salvador Dalí, such as the painting *Leda atómica*.

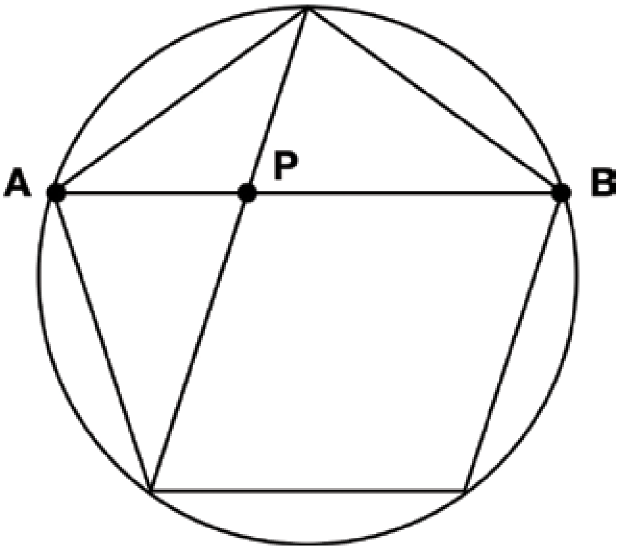

Fig 7. Construction for finding the Golden Ratio
Photo: João Bernardo dos Santos

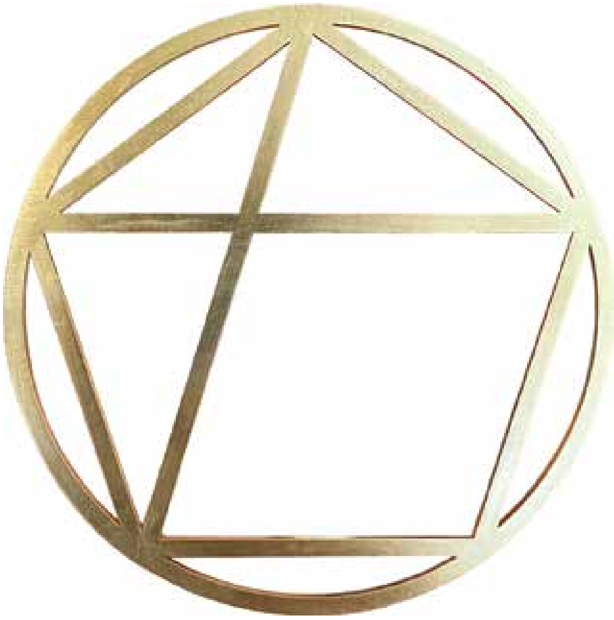

Fig 8. Divina Razão, 2020
João Bernardo dos Santos
Brass, fabricated, 90 mm
Photo: João Bernardo dos Santos

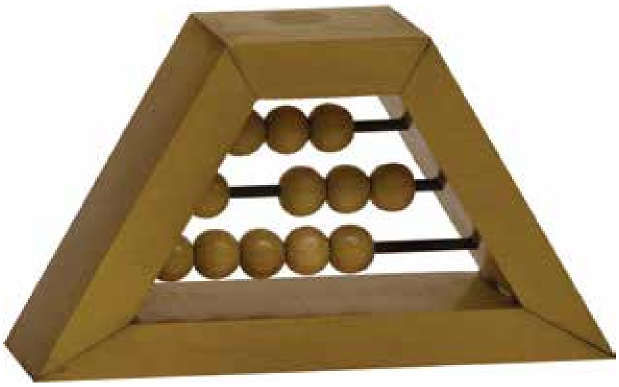

Fig 9. 150.º Aniversário do Banco de Portugal, 1996
João Duarte
Bronze and wood, fabricated, 98 x 55 x 30 mm
Collection: Espaço Volte Face - Medalha
Photo: Espaço Volte Face - Medalha

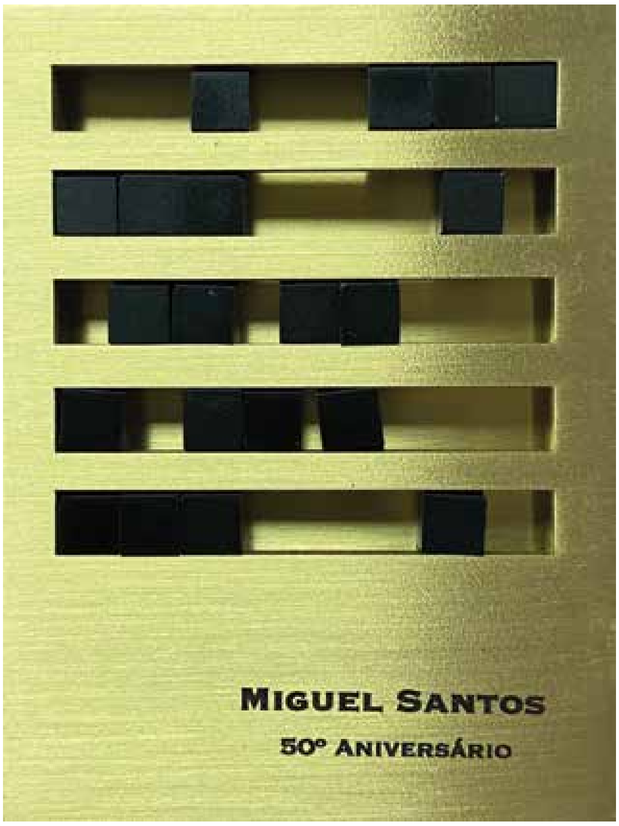

Fig 10. 50.º Aniversário de Miguel Santos, 2019
Brass and acrylic glass, fabricated, 60 x 80 mm
Photo: João Bernardo dos Santos

Figure 7 shows one way of finding the divine proportion. Using the ancient Greek method of straightedge and compass, we draw the circumference, the inscribed regular pentagon[3], and the line segments. The segments $\overline{PB}$ and $\overline{AP}$ are in the divine proportion, meaning $\frac{\overline{PB}}{\overline{AP}} = \varphi$. The medal presented in figure 8 brings this construction to the physical world, and we could use it to calculate an approximate value of phi by measuring those segments and dividing the values. A more accessible way is to divide the longer side by the shorter side of a common credit card.

Regarding calculation, one of its instruments is the ancient, but still in use, abacus. It is a surprisingly very efficient apparatus, especially for summation – an abacus operator famously beat physicist Richard Feynman's pencil and paper. And in the contemporary paradigm of fabricated medals, a working system of beads alluding to the abacus can be an effective symbol for commemorations such as those presented in figures 9 and 10.

Arguably the most important Portuguese mathematician in History – and one of the greatest of his time – was Pedro Nunes (1502–1578).[4] His research was applied in the navigation explorations of the age, and his contributions have been honoured in contemporary medallic art. His work with the *nonius* – a tool he invented to improve the measurements of an astrolabe – has a modern application in callipers, a fact used in his representation on figure 11. Another example in the field of navigation is that he was the first to discuss the rhumb line or loxodrome, which is the arc described by a ship keeping the same angle with the meridians of longitude (fig. 12).

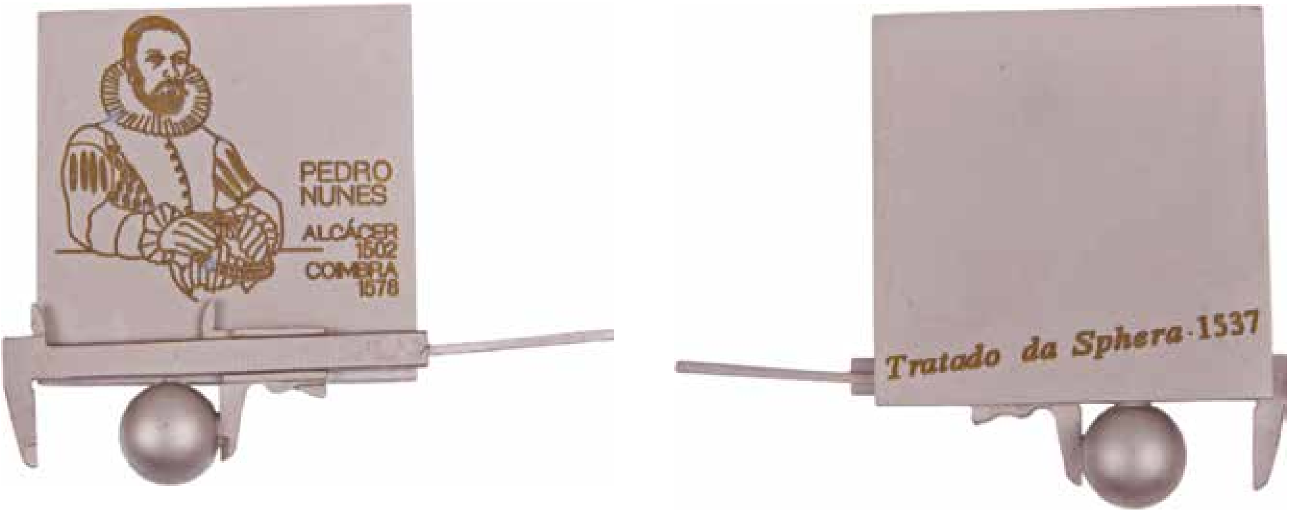

Fig 11. 500th Anniversary of Pedro Nunes, 2002
José João Brito
Nickeled bronze, fabricated, 90 x 70 x 12 mm
Lisbon, Museu Casa da Moeda, MCM 9279
Photo: Museu Casa da Moeda

Fig 12. Loxodrome of Pedro Nunes, 2019
João Bernardo dos Santos
Brass, engraved, 90 mm
Photo: João Bernardo dos Santos

Fig 13. Free hanging chain describing the catenary curve
Photo: João Bernardo dos Santos

On the subject of curves – a dear topic for mathematicians – the last example of this lecture will deal with another one: the curve described by a free hanging rope or chain when suspended between two points (fig. 13). Often confused with a parabola, the catenary – from the latin *catena*, meaning chain – is of completely different mathematical nature. It is a common sight in both natural and urban environments, in such objects as spiderwebs, or overhead telephone and power lines. This curve has the special property of minimising forces in tension, and turned upside down, forming the catenary arch, it minimises the forces in compression. This arch can support its own weight, and has been used extensively in architecture, but is also present in natural formations, such as Rainbow Bridge, in Utah.

Fig 14. Pavillion of Portugal, Lisbon
Photo: Paulo Juntas,
licensed under https://creativecommons.org/licenses/by-sa/2.5/

For the World's Fair of 1998, in Lisbon, architect Álvaro Siza designed the Pavilion of Portugal (fig. 14). The most remarkable feature of the building is the large concrete canopy – inspired by a sheet of paper hanging between two bricks – that describes a catenary curve. For the inauguration of the building, Álvaro Siza also designed

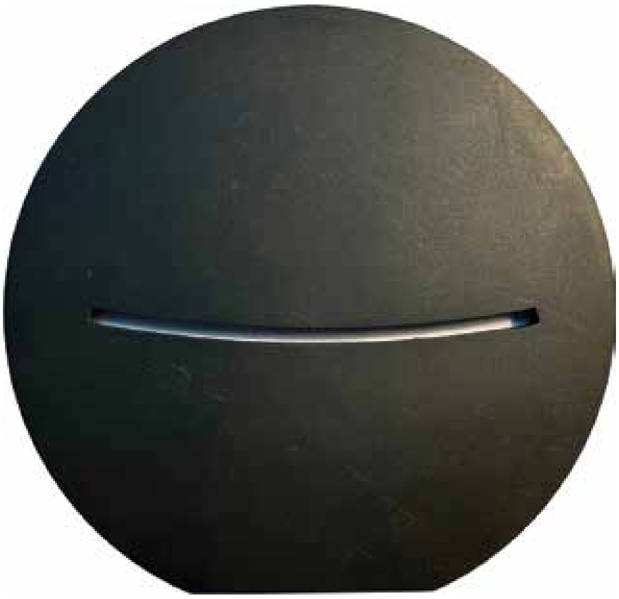

Fig 15. Pavilhão de Portugal – Expo 98, 1998
Álvaro Siza
Bronze, struck, 85 mm
Collection: Espaço Volte Face - Medalha
Photo: João Bernardo dos Santos

a commemorative medal (fig. 15). As the canopy is the most impressive feature of the pavilion, what better motif could there be to feature on the medal than the catenary curve itself?

Producing and analysing a medal means entering a universe which can be contained in a hand, but expands more and more as we gaze into it. It can be an act of discovery and awe. As we have seen, the contemplation and practice of mathematical work can have similar results. Medals are, by design, great vehicles of information, made to spread across time and space. So, when mathematical subjects join in, they can become instruments for the proliferation of reinforced experiences of both intellectual stimulation and aesthetic wonderment.

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NOTES

1. There are no certainties regarding the authorship of each illustration — several artists have been named, such as Titian and Jan van Calcar – but some of the plates are the work of Vesalius himself (Vesalius *et al*., 1973, pp. 25-29).

2. In English: 'We must know, we shall know'.

3. For an animated construction of the regular pentagon inscribed in a circle, refer to, e.g., https://www.mathopenref.com/constinpentagon.html [accessed 1-12-2023].

4. For more on the life and work of Pedro Nunes, refer to: http://pedronunes.fc.ul.pt/biography.html [accessed 1-12-2023].